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Randall-Sundrum Brane Tensions

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ABSTRACT

We show that the singular sources in the energy-momentum tensor for the Randall-Sundrum brane world, viewed as a solution of type IIB supergravity, are composed of two elements. One of these is a D3-brane source with tension opposite in sign to the RS tension in five dimensions; the other arises from patching two regions of flat ten-dimensional spacetime. This resolves an apparent discrepancy between supersymmetry and the sign and magnitude of the RS tension.

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1 Introduction

The original Randall-Sundrum brane-world model [1] (which we will subsequently refer to as Randall-Sundrum I), where our universe is viewed as one of two 3-branes embedded at opposite ends of a patch of five-dimensional anti-de Sitter space, was proposed as a mechanism for naturally generating a large exponential hierarchy based on a small extra dimension. Subsequently, it was realized that the second brane may be pushed off to the Cauchy horizon of the anti-de Sitter space, yielding a model with a single 3-brane and a non-compact extra dimension [2]. Much attention has been drawn to the fact that, contrary to conventional Kaluza-Klein lore, this Randall-Sundrum II model is able to trap gravity to the brane, even with an infinite fifth dimension. This localization of gravity to the brane occurs because the Randall-Sundrum metric takes on a warped product form,

$$ds_5^2 = e^{-2K|z|} \eta_{\mu\nu} dx^\mu dx^\nu + dz^2, \quad (1)$$

with a pronounced “kink” at the brane.¹

Since the Randall-Sundrum geometry, given by the above metric, is essentially that of two Poincaré patches of AdS_5 joined together on a horospherical slice, recent attempts have been made to provide a more stringy origin of the scenario and to relate it to Maldacena’s AdS/CFT conjecture [5, 6, 7]. Hints of this connection were made in [8, 9, 10, 11], where the Randall-Sundrum II model was presented as an anti-de Sitter bulk theory with a boundary not at infinity but instead at some finite location. By pushing the boundary off to infinity, one finally decouples gravity and hence recovers the standard AdS/CFT relation. Further evidence for the equivalence of the two scenarios was presented in [12], which demonstrated that the $1/r^3$ correction the Newtonian potential in the Randall-Sundrum background is reproduced exactly by the one-loop correction to the graviton propagator in the corresponding CFT.

Since AdS_5 arises as the near-horizon geometry of D3-branes in the type IIB theory, it is natural to seek a D3-brane interpretation for the Randall-Sundrum geometry. This connection was made in Refs. [13, 14, 15, 16, 17], which makes use of a massive five-dimensional supergravity scenario originating from an S^5 compactification of type IIB theory with a breathing mode [18]. This result ensures that the Randall-Sundrum scenario may be realized in a fully supersymmetric context. Despite some initial no-go theorems [19, 20] and difficulty in supersymmetrizing the brane-world in a pure $N = 2$ supergravity context [21], it has subsequently been seen that these difficulties are overcome through the incorporation

¹Such types of solutions were first considered in $D = 4$, $N = 1$ supergravity in [3]. For a review, see [4]

of the breathing mode, and by appealing to the higher-dimensional origin of the underlying gauged five-dimensional supergravity [17, 22, 23].

Nevertheless, investigation of the supersymmetric Randall-Sundrum realization [17] has yet to fully resolve the issue of brane tension and the nature of the singular sources for the Randall-Sundrum brane located at $z = 0$. It was pointed out in [24, 17] that the ten-dimensional picture for a Randall-Sundrum II geometry ought to be two segments of positive-energy D3-brane geometry, each carrying N units of charge, with $2N$ D3-branes sitting at the patching location, causing the total charge to add up to zero. It was noted in [17] that if supersymmetry were to be preserved, the $2N$ D3-branes at the patching location would have to have negative tension. However two puzzles arise with this picture of the Randall-Sundrum geometry. Firstly, as shown in [24], the magnitude of the “tension” of the Randall-Sundrum brane [1] is greater than that of a corresponding stack of $2N$ D3-branes.² Secondly, even if the magnitudes of the “tensions” agreed, it is not obvious how negative tension in $D = 10$ could lead to positive tension in $D = 5$.

The resolution to both of these puzzles comes from the realization that the patching of the two stacks of positive tension D3-branes involves more than simply the introduction of $2N$ negative-tension D3-branes to absorb the charge. In joining together two spacetime patches on a curved boundary (which is what occurs in this construction), one must be careful to introduce an appropriate source of curvature in order to satisfy the Israel junction conditions. By careful examination of the Randall-Sundrum metric lifted to $D = 10$, we find that there are in fact two sources of such curvature. The first is of course the stack of the $2N$ negative tension D3-branes, and the second arises from the Z_2 identification of transverse space (which remains present even in the limit when the D3-branes are removed). This dual-source nature of the Randall-Sundrum brane becomes more apparent if the two sources are separated rather than superimposed.

To study this, we begin in section 2 by considering the Randall-Sundrum brane in $D = 5$, and its lifting to $D = 10$. We find that the singular energy-momentum sources in $D = 10$ do not correspond purely to a D3-brane, but instead they have a second contribution coming from the Z_2 identification of flat space. The picture becomes clearer if the five-dimensional solution is generalised to include the breathing mode, since then the ten-dimensional solution describes actual D3-branes rather than merely their near-horizon limit. We therefore extend our discussion of the ten-dimensional singular sources to cover this more general class of solution. A general feature both for these solutions, and for the pure Randall-Sundrum limit,

²This was in fact initially used as an argument for why the Randall-Sundrum brane could not be supersymmetric.

is that the positive-energy brane wall at $z = 0$ turns out to be the sum of a negative-tension D3-brane superimposed on a positive-energy singular source corresponding to making the Z_2 identification of flat spacetime. This latter term outweighs the negative contribution from the D3-brane, and so the net energy is still positive in ten dimensions. We then show how in detail how the tension of the D3-brane agrees perfectly, both in magnitude and sign, with the expectation based upon a careful application of the arguments in [24].

In section 3, we identify and separate the D3-brane Z_2 -identification sources for both the Randall-Sundrum I and II scenarios. In section 4, we explore the behavior of the massless graviton wave function for the case of separated sources. We end with conclusions and some speculations in section 5, including a possible interpretation of the Z_2 -identification sources in terms of averaged arrays of D7-branes. This might give a relation to the F-theory picture described in [25].

2 The brane tension in $D = 10$

In this section, we shall consider the singular sources in $D = 10$ for the Randall-Sundrum model itself (*i.e.* two pieces of AdS_5 glued together) and also a family of generalisations involving the massive breathing-mode scalar field in $D = 5$. We shall postpone the discussion of these generalisations until the next subsection, and shall concentrate first, for simplicity, on the pure Randall-Sundrum case, since this example already illustrates all the essential points.

2.1 The singular sources for the Randall-Sundrum solution

In horospherical coordinates, the Randall-Sundrum solution takes the form (1), which in the bulk is a solution of the five-dimensional Einstein equation with a cosmological constant. The absolute-value symbol on the z in the exponential implies that there will be a delta-function in the curvature, and in fact the vielbein components of the Ricci tensor are given by

$$\begin{aligned} R_{\mu\nu} &= -4K^2 \eta_{\mu\nu} + 2K \delta(z) \eta_{\mu\nu} , \\ R_{55} &= -4K^2 + 8K \delta(z) . \end{aligned} \tag{2}$$

From this it follows that the Einstein tensor $G_{mn} \equiv R_{mn} - \frac{1}{2}R \eta_{mn}$ is given by

$$\begin{aligned} G_{\mu\nu} &= 6K^2 \eta_{\mu\nu} - 6K \delta(z) \eta_{\mu\nu} , \\ G_{55} &= 6K^2 . \end{aligned} \tag{3}$$

The structure of the singular terms in (3), and, in particular, the absence of any singularity in G_{55} , is suggestive of the energy-momentum tensor for a 3-brane source. Note that the Randall-Sundrum Lagrangian,

$$e^{-1}\mathcal{L} = R - \Lambda - \tau_{\text{RS}} (g_{55})^{-1/2} \delta(z), \quad (4)$$

yields the Einstein equation

$$G_{MN} = -\frac{1}{2}\Lambda \eta_{MN} - \frac{1}{2}\tau_{\text{RS}} \eta_{\mu\nu} \delta_M^\mu \delta_N^\nu (g_{55})^{-1/2} \delta(z). \quad (5)$$

Comparing (5) with (3) then fixes the Randall-Sundrum “tension” to be $\tau_{\text{RS}} = 12K$ and the cosmological constant to be $\Lambda = -12K^2$ [1, 2].

However, it should be borne in mind that there is no fundamental 3-brane as such in $D = 5$, and so we should really reserve judgment about how to interpret the singular terms and the meaning of τ_{RS} until we have lifted the solution back to $D = 10$ type IIB supergravity. This is easily done; the only non-vanishing ten-dimensional fields will be the metric and the self-dual 5-form, given by

$$\begin{aligned} d\hat{s}_{10}^2 &= ds_5^2 + m^{-2} d\Omega_5^2, \\ \hat{F}_{(5)} &= 4m \epsilon_{(5)} + 4m^{-4} \Omega_{(5)}, \end{aligned} \quad (6)$$

where $K^2 = m^2$, ϵ_5 is the volume form of the metric $ds_{(5)}^2$ given in (1), and $\Omega_{(5)}$ is the volume form of the unit S^5 metric $d\Omega_5^2$. In fact we should choose

$$m = \begin{cases} +K, & z > 0, \\ -K, & z < 0, \end{cases} \quad (7)$$

in order to ensure continuity of the Killing spinors across the junction at $z = 0$ [17].

It is now a simple matter using (2) to calculate the Ricci tensor \hat{R}_{AB} , and hence the Einstein tensor \hat{G}_{AB} , in $D = 10$. We find that the vielbein components are given by

$$\begin{aligned} \hat{G}_{\mu\nu} &= -4K^2 \eta_{\mu\nu} - 6K \delta(z) \eta_{\mu\nu}, \\ \hat{G}_{55} &= -4K^2, \\ \hat{G}_{ab} &= 4K^2 \delta_{ab} - 8K \delta(z) \delta_{ab}. \end{aligned} \quad (8)$$

Naively, by looking just at the $\hat{G}_{\mu\nu}$ and \hat{G}_{55} components, we might be tempted to interpret the singularities as being due to a D3-brane source with tension proportional to

$$\tilde{\sigma} = 12K. \quad (9)$$

However, it is evident from the fact that there are singular terms also in the S^5 directions \hat{G}_{ab} that we cannot simply attribute the singularities to a D3-brane source, which would have the structure

$$\begin{aligned}\hat{T}_{\mu\nu}^{\text{D3}} &= -\frac{1}{2}\tilde{\sigma}\delta(z)\eta_{\mu\nu}, \\ \hat{T}_{55}^{\text{D3}} &= 0, \\ \hat{T}_{ab}^{\text{D3}} &= 0.\end{aligned}\tag{10}$$

The precise relation between $\tilde{\sigma}$ and the D3-brane tension τ_{D3} may be determined by considering the $D = 10$ supergravity coupled D3-brane action

$$S_{10} = \int d^{10}x \sqrt{-\hat{g}} [\hat{R} + \cdots] - \tau_{\text{D3}} \int d^4\xi \sqrt{-\bar{g}} + \mu_{\text{D3}} \int_{\mathcal{M}_4} \hat{A}_{[4]} \tag{11}$$

(for vanishing gauge fields on the D3-brane) where \bar{g} is the induced metric on the brane. In physical gauge, the resulting stress tensor for a single D3-brane source has the form

$$\hat{T}_{MN}^{\text{D3}} = -\frac{1}{2}\tau_{\text{D3}}\eta_{\mu\nu}\delta_M^\mu\delta_N^\nu\frac{\delta^6(\vec{y}-\vec{y}_0)}{\sqrt{\det\hat{g}_{ij}}}, \tag{12}$$

where \hat{g}_{ij} is the metric in the space transverse to the brane, which is located at \vec{y}_0 . To enforce the S^5 symmetry in (6), one must take a spherical distribution of branes. For a total of N D3-branes in the above geometry, the stress tensor becomes

$$\hat{T}_{MN}^{\text{D3}} = -\frac{1}{2}N K^5 \pi^{-3} \tau_{\text{D3}} \eta_{\mu\nu} \delta_M^\mu \delta_N^\nu \delta(z), \tag{13}$$

where $K^{-5}\pi^3$ is the volume of S^5 . From the ten-dimensional metric in (6), we therefore find that the coefficient $\tilde{\sigma}$ in (10) is related to the fundamental D3-brane tension by $\tilde{\sigma} = N K^5 \pi^{-3} \tau_{\text{D3}}$.

In fact the singularities in the Einstein tensor (8) can be understood as coming from two distinct sources. One contribution is indeed a fundamental D3-brane, while the other is a contribution that would arise even in flat space, in the absence of any D3-brane, as a result of having performed the Z_2 identification implied by the use of the absolute-value symbol on z in (1). It is easier to understand this second contribution in the more general context of the breathing-mode solutions that we shall discuss below. However, in order to be able to complete our present discussion for the AdS_5 solution we shall just quote for now from some results that we shall derive later. We find that the ten-dimensional Einstein tensor that results from the Z_2 identification of flat space has singularities that are supplied by an energy-momentum tensor of the following structure:

$$\hat{T}_{\mu\nu}^{Z_2} = -\frac{1}{2}\tilde{\kappa}\delta(z)\eta_{\mu\nu},$$

$$\begin{aligned}\hat{T}_{55}^{Z_2} &= 0, \\ \hat{T}_{ab}^{Z_2} &= -\frac{2}{5}\tilde{\kappa}\delta(z)\delta_{ab}.\end{aligned}\tag{14}$$

It is the *sum* $\hat{T}_{AB}^{\text{D3}} + \hat{T}_{AB}^{Z_2}$ that should be matched to the singular terms in the Einstein tensor (8). Clearly we shall have $\tilde{\kappa} = 20K$, and so instead of the D3-brane tension being given by (9), it is actually given by

$$\tilde{\sigma} = -8K.\tag{15}$$

Not only is the correct tension smaller by a factor of $\frac{2}{3}$ than the “naive” value [24], it is also of the opposite sign! Thus a Randall-Sundrum domain wall in five dimensions that would conventionally be described as having “positive tension” is actually supported, from the ten-dimensional viewpoint, by a negative-tension D3-brane source together with a superimposed source term associated with the Z_2 identification of flat space. The latter contribution outweighs the former, so that the sign of the total \hat{T}_{00} contribution is still positive in $D = 10$.

At this point it is useful to revisit the tension argument of [24], which relates the $D = 10$ D3-brane tension to $D = 5$ brane-world quantities. As indicated in [24, 17], the supersymmetric Randall-Sundrum realization has the brane-world sitting between two stacks of N D3-branes. The AdS curvature is then related to N by [5]

$$4\pi^3 K^{-4} = N \tau_{\text{D3}}.\tag{16}$$

Based on flux conservation, the Randall-Sundrum brane must then carry $-2N$ units of charge. In order for this configuration to be supersymmetric, however, this negative charge must be accompanied by negative tension. Using the previously derived relation between $\tilde{\sigma}$ and τ_{D3} , the corresponding tension of $2N$ negatively-charged D3-branes is then [24]

$$\tilde{\sigma} = -2N K^5 \pi^{-3} \tau_{\text{D3}} = -8K,\tag{17}$$

which agrees with (15), and thus confirms the above split of \hat{T}_{MN} into a D3-brane source and the Z_2 identification of flat space.

Although this result, especially the change of sign of the D3-brane tension, might seem a little surprising, it is actually rather natural from the ten-dimensional viewpoint. It is rather easier to understand in the context of the more general family of Randall-Sundrum type solutions that make use of the breathing-mode scalar in five dimensions, and it is to this subject that we now turn.

2.2 Singular sources for the breathing-mode solutions

Here, we consider generalisations of the Randall-Sundrum solution, namely certain 3-brane solutions in five dimensions that are supported by a subset of the complete set of fields that come from the S^5 reduction of type IIB supergravity. Specifically, the relevant fields are the metric tensor and the breathing-mode scalar that parameterises the overall volume of the 5-sphere. The solutions were first obtained in [18], and the idea of using them in the context of a Randall-Sundrum II scenario was introduced in [13], and studied further in [14, 15]. The reason for considering the breathing-mode scalar was that it is a massive field (lying outside the massless supergravity multiplet that is commonly considered), and its potential has a minimum rather than a maximum. It was shown in [14] that although it still cannot give rise to a smooth gravity-trapping domain wall, it can give an acceptable singular wall of the Randall-Sundrum II type if two segments of the solution are appropriately patched with a delta-function in the curvature at the “join.” The same bulk solution was then used in [17] for the construction of a Randall-Sundrum I type of scenario, with two singular branes, one of positive and the other of negative tension.

To study the brane tensions in detail, we first need to review the pertinent aspects of the S^5 reduction of type IIB supergravity to obtain the theory in $D = 5$ with the breathing mode, and to find the brane solution in five dimensions. These results are taken from [18].

The only non-vanishing fields in the type IIB theory are the metric \hat{g}_{MN} and the self-dual 5-form $\hat{F}_{(5)}$. The type IIB field equations $\hat{R}_{MN} = \frac{1}{96}\hat{F}_{MN}^2$, $\star\hat{F}_{(5)} = \hat{F}_{(5)}$ and $d\hat{F}_{(5)} = 0$ are satisfied by the Ansatz

$$\begin{aligned} d\hat{s}_{10}^2 &= e^{\frac{1}{2}\sqrt{\frac{5}{3}}\varphi} ds_5^2 + e^{-\frac{1}{2}\sqrt{\frac{3}{5}}\varphi} ds^2(S^5), \\ \hat{F}_{(5)} &= 4m e^{2\sqrt{\frac{5}{3}}\varphi} \epsilon_{(5)} + 4m \epsilon(S^5), \end{aligned} \quad (18)$$

provided that the five-dimensional fields ds_5^2 and φ satisfy the equations of motion following from the Lagrangian

$$e^{-1}\mathcal{L}_5 = R - \frac{1}{2}(\partial\varphi)^2 - 8m^2 e^{2\sqrt{\frac{5}{3}}\varphi} + e^{\frac{4}{\sqrt{15}}\varphi} R_5. \quad (19)$$

Here $\epsilon_{(5)}$ is the volume form of ds_5^2 , $\epsilon(S^5)$ is the volume form of the metric $ds^2(S^5)$ on the 5-sphere, which has (constant) Ricci scalar R_5 , and m is another constant. It is useful to note that the vielbein components of the ten-dimensional Ricci tensor for the metric $d\hat{s}_{10}^2$ in (18) are given by

$$\hat{R}_{mn} = e^{-\frac{1}{2}\sqrt{\frac{5}{3}}\varphi} \left(R_{mn} - \frac{1}{4}\sqrt{\frac{5}{3}}\square\varphi\eta_{mn} - \frac{1}{2}\nabla_m\varphi\nabla_n\varphi \right),$$

$$\begin{aligned}\hat{R}_{ab} &= e^{\frac{1}{2}\sqrt{\frac{3}{5}}\varphi} R_{ab} + \frac{1}{4}\sqrt{\frac{3}{5}} e^{-\frac{1}{2}\sqrt{\frac{5}{3}}\varphi} \square\varphi \delta_{ab}, \\ \hat{R}_{ma} &= 0,\end{aligned}\tag{20}$$

where the index m lies in the five-dimensional spacetime, and a lies in S^5 .

The relevant domain-wall solution of the five-dimensional equations of motion is given by [18]

$$\begin{aligned}ds_5^2 &= e^{2A} dx^\mu dx_\mu + e^{-8A} dz^2, \\ e^{-\frac{7}{\sqrt{15}}\varphi} &= H = e^{-\frac{7}{\sqrt{15}}\varphi_0} + k|z|, \\ e^{4A} &= \tilde{b}_1 H^{\frac{2}{7}} + \tilde{b}_2 H^{\frac{5}{7}},\end{aligned}\tag{21}$$

where $\tilde{b}_1 = \mp 28m/(3k)$ and $\tilde{b}_2 = \pm 14\sqrt{5R_5}/(15k)$. Of course, by placing the absolute-value symbol around the coordinate z we have caused delta-functions to appear in the expressions for the Ricci tensor and $\square\varphi$, and so while (21) solves the five-dimensional equations of motion exactly when $z \neq 0$, there will be a need for singular source terms on the domain wall itself.

Solutions exist for all four independent choices of signs for \tilde{b}_1 and \tilde{b}_2 , but the ones that are relevant here have $\tilde{b}_1 \tilde{b}_2 < 0$. The AdS₅ limit is reached by sending k to zero, with the constant φ_0 given by

$$e^{\frac{6}{\sqrt{15}}\varphi_0} = \frac{R_5}{20m^2}.\tag{22}$$

The solution (21) in this limit has

$$e^{4A} = 4m e^{\frac{5}{\sqrt{15}}\varphi_0} (z_0 - |z|), \quad \varphi = \varphi_0,\tag{23}$$

and can be seen, after a simple coordinate transformation, to be equivalent to the AdS₅ solution (1). The constant K appearing in (1) and (2) is given by

$$K = m \left(\frac{R_5}{20m^2} \right)^{5/6}.\tag{24}$$

Since our purpose is to study the precise nature of the singular sources, we shall not need to present the “regular” terms in what follows, since it is already established from the results of [18] that the five-dimensional and ten-dimensional equations are satisfied in the bulk. We shall therefore generally simply represent the regular terms by “Reg” in the equations. A straightforward calculation from (21) shows that $\square\varphi$ and the vielbein components of the Ricci tensor take the forms [17]

$$\begin{aligned}\square\varphi &= -\frac{2\sqrt{15}}{7} c^{-7} k (\tilde{b}_1 c^2 + \tilde{b}_2 c^5) (g_{55})^{-1/2} \delta(z) + \text{Reg}, \\ R_{\mu\nu} &= -\frac{1}{14} k (2\tilde{b}_1 c^{-5} + 5\tilde{b}_2 c^{-2}) (g_{55})^{-1/2} \delta(z) \eta_{\mu\nu} + \text{Reg}, \\ R_{55} &= -\frac{2}{7} k (2\tilde{b}_1 c^{-5} + 5\tilde{b}_2 c^{-2}) (g_{55})^{-1/2} \delta(z) + \text{Reg},\end{aligned}\tag{25}$$

where for convenience we have defined the constant c by

$$c \equiv e^{-\frac{1}{\sqrt{15}} \varphi^0}. \quad (26)$$

Note that the five-dimensional Einstein tensor is therefore given by

$$\begin{aligned} G_{\mu\nu} &= \frac{3}{14} k (2\tilde{b}_1 c^{-5} + 5\tilde{b}_2 c^{-2}) (g_{55})^{-1/2} \delta(z) \eta_{\mu\nu} + \text{Reg}, \\ G_{55} &= 0 + \text{Reg}. \end{aligned} \quad (27)$$

These singular terms have the general structure one would expect of a 3-brane source in $D = 5$. However, as in our previous discussion for the AdS_5 case, the absence of a fundamental 3-brane in $D = 5$ means that we should postpone interpreting the singular terms until we have lifted the solution back to $D = 10$. It is known that the bulk solution (21) oxidises to give a standard D3-brane supergravity solution in $D = 10$ [18], namely

$$d\hat{s}_{10}^2 = \tilde{H}(r)^{-\frac{1}{2}} d\tilde{x}^\mu d\tilde{x}_\mu + \tilde{H}(r)^{\frac{1}{2}} (dr^2 + r^2 d\Omega_5^2), \quad (28)$$

where

$$\tilde{H}(r) \equiv 1 + \frac{\tilde{k}}{r^4}, \quad (29)$$

and

$$\tilde{x}^\mu = \tilde{b}_2^{1/2} x^\mu, \quad r^4 = (20/R_5)^2 H^{\frac{3}{7}} - \tilde{k}, \quad \tilde{k} = -(20/R_5)^2 \frac{\tilde{b}_1}{\tilde{b}_2}, \quad (30)$$

as can be seen by substituting (21) into (18). One might be tempted to expect that after lifting the source terms to $D = 10$, they would become precisely those for the D3-brane. As we shall now show, this is not the case.

It is a straightforward matter to lift the five-dimensional expressions (25) back to $D = 10$, using the reduction Ansatz (18). In particular, using (20) we can calculate the vielbein components of the ten-dimensional Einstein tensor, for the oxidation of the domain-wall solution (21) to $D = 10$, finding

$$\begin{aligned} \hat{G}_{\mu\nu} &= \frac{3}{14} (2\tilde{b}_1 + 5\tilde{b}_2 c^3) c^{-15/4} k (\hat{g}_{55})^{-1/2} \delta(z) \eta_{\mu\nu} + \text{Reg}, \\ \hat{G}_{55} &= 0 + \text{Reg}, \\ \hat{G}_{ab} &= \frac{6}{7} \tilde{b}_2 c^{-3/4} k (\hat{g}_{55})^{-1/2} \delta(z) \delta_{ab} + \text{Reg}. \end{aligned} \quad (31)$$

Clearly this is not of the form of a pure 3-brane source in $D = 10$, which would have the structure given in (10).

As we have already foreshadowed, the resolution of the puzzle is that the singular terms we are seeing in (31) are actually accounted for by the sum of two quite distinct singular

sources. One of these is a D3-brane source, and the other describes the curvature singularities that result from identifying two patches of flat spacetime under Z_2 . In fact the situation is made unnecessarily confusing by having these two sources located at the same value of the coordinate z (or r) transverse to the brane, and later on we shall find it convenient to separate the two contributions and locate them at different values of r .

To recognise how the singular sources for (31) separate into the two contributions we can either first isolate the D3-brane contribution, after which the remainder must be the term describing the Z_2 identification of flat space, or else we can first isolate the contribution from the Z_2 identification. Although it is perhaps logically more natural to follow the former approach, calculationally it seems to be simpler first to calculate the curvature singularities for Z_2 -identified flat space, and then after extracting this part from (31); the remainder will be the D3-brane contribution.

To calculate the Ricci curvature for Z_2 -identified flat space, it is simplest to perform the computation in $D = 5$, and then lift the results back to $D = 10$. We begin by picking a radius $r = r_0$ in (28), and then taking the flat-space metric that continuously matches at this radius, namely

$$d\hat{s}_{10}^2 = \tilde{H}(r_0)^{-\frac{1}{2}} d\tilde{x}^\mu d\tilde{x}_\mu + \tilde{H}(r_0)^{\frac{1}{2}} (dr^2 + r^2 d\Omega_5^2), \quad (32)$$

We now reduce this to $D = 5$, using the reduction Ansatz (18). At the same time, we make the following redefinitions

$$r_0^4 + \tilde{k} = c^3, \quad r^4 = r_0^4 + k |\xi|. \quad (33)$$

Thus by using the ξ coordinate, we have arranged that as ξ passes from negative to positive, r approaches r_0 , and then retraces its steps. This introduces a “join” at $r = r_0$. In terms of z , the $D = 5$ metric becomes

$$\begin{aligned} ds_5^2 &= r_0^{-\frac{4}{3}} c (r_0^4 + k |\xi|)^{\frac{5}{6}} dx^\mu dx_\mu + \frac{1}{16} k^2 r_0^{-\frac{16}{3}} c^4 (r_0^4 + k |\xi|)^{-\frac{2}{3}} d\xi^2, \\ e^{-\sqrt{\frac{3}{5}}\varphi} &= r_0^{-4} c^3 (r_0^4 + k |\xi|). \end{aligned} \quad (34)$$

We then find that

$$\begin{aligned} R_{\mu\nu} &= \frac{1}{6} \tilde{\kappa} c^{-\frac{5}{4}} (g_{55})^{-1/2} \delta(\xi) \eta_{\mu\nu} + \text{Reg}, \\ R_{55} &= \frac{2}{3} \tilde{\kappa} c^{-\frac{5}{4}} (g_{55})^{-1/2} \delta(\xi) + \text{Reg}, \\ \frac{1}{4} \sqrt{\frac{5}{3}} \square \varphi &= \frac{1}{6} \tilde{\kappa} c^{-\frac{5}{4}} (g_{55})^{-1/2} \delta(\xi) + \text{Reg}, \end{aligned} \quad (35)$$

where

$$\tilde{\kappa} \equiv -20 c^{-\frac{3}{4}}. \quad (36)$$

Lifting to $D = 10$, we find that the Einstein tensor for Z_2 -identified flat space has the following vielbein components:

$$\begin{aligned}\hat{G}_{\mu\nu} &= -\frac{1}{2}\tilde{\kappa}(\hat{g}_{55})^{-1/2}\delta(\xi)\eta_{\mu\nu}, \\ \hat{G}_{55} &= 0, \\ \hat{G}_{ab} &= -\frac{2}{5}\tilde{\kappa}(\hat{g}_{55})^{-1/2}\delta(\xi)\delta_{ab}.\end{aligned}\tag{37}$$

Since this is not a brane-source term it is inappropriate to speak of a “tension” associated with this contribution. We shall merely describe it as having positive energy if $\tilde{\kappa}$ is positive, and *vice versa*.

Using this result, we can now make the unique decomposition of the singular terms in the ten-dimensional Einstein tensor (31) into the sum of a D3-brane source (10) and the identified flat-space source (37). Thus we find

$$\tilde{\sigma} = -\frac{6k}{7}c^{-15/4}\tilde{b}_1, \quad \tilde{\kappa} = -\frac{15k}{7}c^{-3/4}\tilde{b}_2.\tag{38}$$

Using $\tilde{b}_1 = 28m/(3k)$ allows us to rewrite the D3-brane tension as $\tilde{\sigma} = -8mc^{-15/4}$. Now, in deriving the fundamental D3-brane tension from (12), one has to account for the complete Ansatz (18) when determining the volume of S^5 . The resulting relation is

$$\tilde{\sigma}(\text{single D3-brane}) = \left(\frac{R_5}{20}\right)^{5/2}\pi^{-3}c^{-15/4}\tau_{\text{D3}}.\tag{39}$$

At the same time, the AdS relation, (16), is modified to³

$$4\pi^3 m \left(\frac{R_5}{20}\right)^{-5/2} = N\tau_{\text{D3}}.\tag{40}$$

Thus one finds simply

$$\tilde{\sigma} = -2N\tilde{\sigma}(\text{single D3-brane}),\tag{41}$$

which once again corresponds to having a source of $2N$ negative tension D3-branes.

It is clear that the “tension” $\tilde{\kappa}$ arising from the Z_2 identification cannot be identified with any conventional brane source. After all, a p -brane with tension τ_p would yield a contribution to the $D = 10$ stress tensor of the form

$$\hat{T}_{MN}^p = -\frac{1}{2}\tau_p\eta_{\mu\nu}\delta_M^\mu\delta_N^\nu\frac{\delta^{9-p}(\vec{y}-\vec{y}_0)}{\sqrt{\det\hat{g}_{ij}}},\tag{42}$$

where \hat{g}_{ij} is the metric transverse to the brane. Unlike (37), this p -brane contribution is isotropic in all directions longitudinal to the brane.

³This relation may be derived by comparing the factor \tilde{k} in the D3-brane harmonic function, (29), with that demanded by charge quantization.

3 Separating the singular sources

Having seen that the singular terms in the Einstein tensor in $D = 10$ are actually matched by two quite distinct sources, namely a D3-brane and the term associated with the Z_2 identification of flat space, we see that it now becomes natural to dissociate the two terms from one another, by placing them at different values of r in the ten-dimensional solution. We shall discuss this first for the case of a Randall-Sundrum II type scenario, where there is just a single domain wall in five dimensions, at $z = 0$. After doing this, we shall then discuss the case of a Randall-Sundrum I model, where a second domain wall is introduced at $z = L$, by identifying the coordinate values $z = +L$ and $z = -L$.

3.1 Ten-dimensional sources for a single domain wall

It can be seen from (30) that as z ranges from $z = 0$ on the five-dimensional domain wall to $z = \pm\infty$ on the Cauchy horizon, the radial coordinate r in the ten-dimensional D3-brane solution ranges from some fixed value $r > 0$ outside the horizon to $r = 0$, on the D3-brane horizon. We shall now deform the geometry so that the S^5 spherical shell of D3-brane source and the Z_2 -identification source are located at two different values of r . Specifically, as r increases from the horizon at $r = 0$, the D3-brane source will be encountered first, at some radius $r = r_1$. We saw by direct calculation that the D3-brane tension was coming out to be negative, and in fact this is very natural. It has the effect of “turning off” the D3-brane charge for values of r that lie outside $r = r_1$, so that for $r > r_1$ the D3-brane solution (28) is replaced by a flat spacetime. Of course the metrics must match continuously at $r = r_1$, and so we shall have that for $r > r_1$, the flat metric is given by

$$d\hat{s}_{10}^2 = \tilde{H}(r_1)^{-\frac{1}{2}} d\tilde{x}^\mu d\tilde{x}_\mu + \tilde{H}(r_1)^{\frac{1}{2}} (dr^2 + r^2 d\Omega_5^2), \quad (43)$$

It should be remarked that one can explicitly check, using the relations given in (30), that the D3-brane tension σ that we calculated in (38) is precisely the correct value to “turn off” the D3-brane (28) for r lying outside the radius r_1 at which we have now chosen to locate the D3-brane source.

Proceeding outwards, we now choose a radius $r_0 > r_1$ at which the Z_2 identification of the flat spacetime (43) will be performed. This implies a singular source term of the form (37), with positive energy. The ten-dimensional solution is depicted in Figure 1.

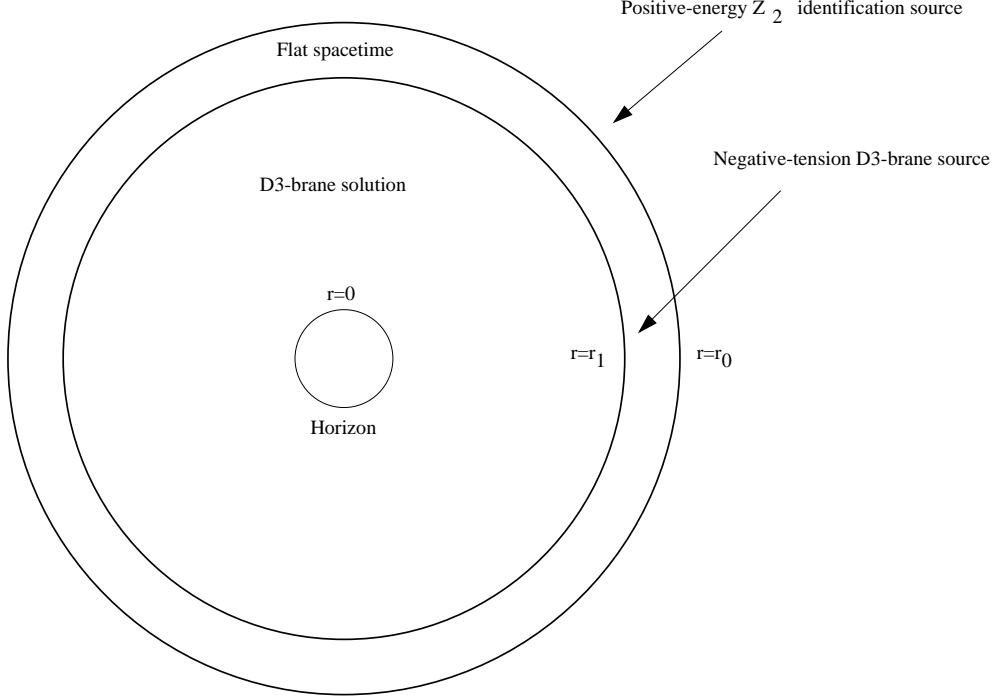


Figure 1: The ten-dimensional configuration for the RS II brane

3.2 Ten-dimensional sources for two domain walls

The situation described above is for a single domain wall in five dimensions, corresponding to a model of the Randall-Sundrum II type. We may also consider the situation where a second domain wall is introduced at $z = L$, by identifying $z = L$ with $z = -L$. This is conventionally described, in five dimensions, as having “negative tension.” Again, in ten dimensions we shall separate the singular terms associated with this second wall into a D3-brane source and a Z_2 -identification source, located at two different values of r . Since its five-dimensional “tension” is of the opposite sign to that of the wall at $z = 0$, we know from our earlier results that the associated D3-brane source in $D = 10$ will have positive tension. The second wall lies at a smaller radius r than the original one. Thus we shall now have a total of four radii at which singular sources are located, which we take to be in the order $r_3 < r_2 < r_1 < r_0$. The disposition of sources will be as follows. At $r = r_0$ is the Z_2 -identification source for the $z = 0$ domain wall, and at $r = r_1$ is the negative-tension D3-brane source for the $z = 0$ domain wall, exactly as in the previous subsection. Moving inwards, we next encounter a positive-tension D3-brane source for the $z = L$ domain wall, at $r = r_2$. Finally, we encounter a Z_2 -identification source at $r = r_3$, which has negative energy.

The region $r_2 \leq r \leq r_1$ has the form of a solid annulus of the D3-brane solution (28).

Outside this, for $r_1 \leq r \leq r_0$, and inside it, for $r_3 \leq r \leq r_2$, are solid annuli of flat spacetime. The ten-dimensional spacetime is depicted in Figure 2.

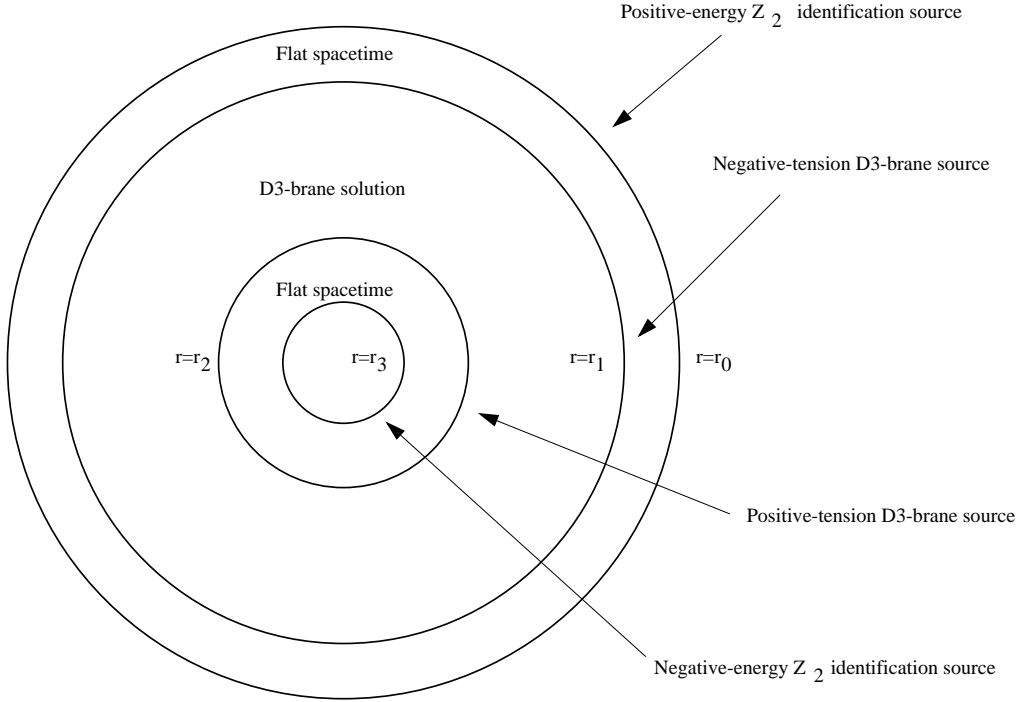


Figure 2: The ten-dimensional configuration for the RS I branes

In this situation with two domain walls, the effect of the Z_2 identification is as follows. The coordinate z can be thought of as a coordinate on the circle S^1 , with $-L \leq z \leq L$, which is then factored by Z_2 in the style of Horava and Witten. At the same time, as one crosses from negative to positive z the sign of the parameter m in the expression for $\hat{F}_{(5)}$ in (18) reverses, corresponding to a reversal of the orientation of the compactifying 5-sphere. Thus the ten-dimensional theory is reduced on the orbifolded internal manifold $(S^1 \times S^5)/Z_2$.

4 The wave function for the massless graviton mode

In the conformally-flat frame

$$ds_5^2 = e^{2A(z)} (dx^\mu dx_\mu + dz^2), \quad (44)$$

the massless wave function is just given by

$$\psi(z) = e^{\frac{3}{2}A(z)}. \quad (45)$$

We can study this explicitly for the case where we take the pure AdS₅ limit. In fact it is easier then to derive the result directly, rather than to get it by explicitly taking the singular $k \rightarrow 0$ limit. But we still want to separate the D3-brane and Z_2 -identification sources. Accordingly, we consider the ten-dimensional metrics in two regimes:

$$\begin{aligned} \underline{r \leq r_1} : \quad d\hat{s}_{10}^2 &= \frac{r^2}{r_1^2} dx^\mu dx_\mu + \frac{r_1^2}{r^2} dr^2 + r_1^2 d\Omega_5^2, \\ \underline{r \geq r_1} : \quad d\hat{s}_{10}^2 &= dx^\mu dx_\mu + dr^2 + r^2 d\Omega_5^2. \end{aligned} \quad (46)$$

This is the AdS₅ solution, for $r \leq r_1$, obtained by taking the near-horizon limit in which the “1” is dropped in the harmonic function $\tilde{H}(r) = 1 + \tilde{k}/r^4$, joined on continuously to pure flat space for $r \geq r_1$.

We now reduce to $D = 5$, using the standard reduction Ansatz

$$d\hat{s}_{10}^2 = e^{2\alpha\varphi} ds_5^2 + e^{-\frac{6}{5}\alpha\varphi} d\Omega_5^2. \quad (47)$$

Thus for the two regimes we have

$$\begin{aligned} \underline{r \leq r_1} : \quad ds_5^2 &= \frac{r^2}{r_1^2} dx^\mu dx_\mu + \frac{r_1^2}{r^2} dr^2, \quad \varphi = 0, \\ \underline{r \geq r_1} : \quad ds_5^2 &= \left(\frac{r}{r_1}\right)^{\frac{10}{3}} (dx^\mu dx_\mu + dr^2), \quad e^{-\frac{6}{5}\alpha\varphi} = \frac{r^2}{r_1^2}. \end{aligned} \quad (48)$$

We now change to the z coordinate, which transforms the metric into the conformally-flat form (44). In the two regimes we therefore have

$$\begin{aligned} \underline{r \leq r_1} : \quad \frac{r_1}{r} &= 1 + \frac{z - z_1}{r_1}, \\ \underline{r \geq r_1} : \quad \frac{r}{r_1} &= 1 + \frac{z_1 - z}{r_1}, \end{aligned} \quad (49)$$

where we have chosen the constants of integration for convenience, and so that the join ($r = r_1$) occurs at $z = z_1$.

We now choose a Z_2 identification at $r = r_0 > r_1$. We shall take this to be at $z = 0$, so now we have

$$\begin{aligned} \underline{r \leq r_1} : \quad \frac{r_1}{r} &= 1 + \frac{|z| - z_1}{r_1}, \\ \underline{r_1 \leq r \leq r_0} : \quad \frac{r}{r_1} &= 1 + \frac{z_1 - |z|}{r_1}, \end{aligned} \quad (50)$$

The upper equation corresponds to $|z| \geq z_1$, while the lower corresponds to $|z| \leq z_1$. (We assume, without loss of generality, that z_1 is positive.) Note that we shall have

$$z_1 = r_0 - r_1. \quad (51)$$

To summarise, we now have a positive-energy source at $z = 0$, corresponding to $r = r_0$, which results from the Z_2 identification, and a pair of negative-tension D3-brane sources at $z = \pm z_1$, corresponding to $r = r_1 < r_0$. As the coordinate z goes to the Cauchy horizons at $\pm\infty$, the radial coordinate r tends to zero.

We see from (45) that the wavefunction in the two regimes is given by

$$\begin{aligned} |z| \geq z_1 : \quad \psi(z) &= a e^{\frac{3}{2}A(z)} = a \left[1 + \frac{|z| - z_1}{r_1} \right]^{-\frac{3}{2}}, \\ |z| \leq z_1 : \quad \psi(z) &= a e^{\frac{3}{2}A(z)} = a \left[1 + \frac{z_1 - |z|}{r_1} \right]^{\frac{5}{2}}, \end{aligned} \quad (52)$$

where a is a normalisation constant. It can be determined by requiring that $\int_{-\infty}^{\infty} dz |\psi|^2 = 1$, and this implies

$$a = \frac{\sqrt{3} r_1^{5/2}}{\sqrt{r_0^6 + 2r_1^6}}. \quad (53)$$

Expressed back in terms of the r coordinate, the normalised wavefunction becomes

$$\begin{aligned} 0 \leq r \leq r_1 : \quad \psi &= \frac{\sqrt{3} r_1 r^{3/2}}{\sqrt{r_0^6 + 2r_1^6}}, \\ r_1 \leq r \leq r_0 : \quad \psi &= \frac{\sqrt{3} r^{5/2}}{\sqrt{r_0^6 + 2r_1^6}}. \end{aligned} \quad (54)$$

From this it follows that the wavefunction concentrates around the Z_2 -reflection point at $r = r_0$, even more strongly than in the Randall-Sundrum case where $r_1 = r_0$. Conversely, it tends to be suppressed on the D3-branes at $r = r_1$.

5 Conclusion and speculations

In this paper, we have investigated the lifting of the pure Randall-Sundrum AdS_5 solution, and its breathing-mode generalisations, from $D = 5$ to $D = 10$. Since the five-dimensional breathing-mode solutions are nothing but D3-brane solutions when lifted to $D = 10$, this procedure provides a natural mechanism for interpreting the singular sources in terms of ten-dimensional D3-brane sources. Likewise, the pure Randall-Sundrum AdS_5 limit corresponds to taking the near-horizon limit of the D3-brane in ten dimensions.

After lifting the solutions, we find that the singular sources are not accounted for purely by D3-brane sources in ten dimensions. In fact the singular terms in the energy-momentum tensor can be split into the sum of a standard type of D3-brane source, plus a second term that would be present even in the absence of any D3-branes, which results from performing the Z_2 identification of flat spacetime. Having disentangled the two contributions, we were then able to show that the D3-brane part has exactly the correct magnitude, and sign, to

account for the expected quantity $(-2N)$ of D3-brane charge that should be present at $z = 0$ in order to counterbalance the presence of N D3-branes placed to the left and to the right of the brane wall. In other words, the puzzles that had been raised in [24], which seemed to indicate a mismatch of brane charges, are now fully resolved.

An element of the resolution, the necessity for which was not highlighted in [24], is that not only is the magnitude of the D3-brane charge different from the “naive” value that one might be tempted to read off in $D = 5$, but also the sign is reversed. In particular, this means that what is customarily called a “positive tension brane” in $D = 5$ actually corresponds to a negative-tension D3-brane in $D = 10$. This does not, however, mean that the sign of the energy is reversed on lifting the solution from $D = 5$ to $D = 10$, since the negative D3-brane contribution to the energy in $D = 10$ is actually outweighed by a larger positive contribution associated with the Z_2 identification of flat space.

Having seen that the singular sources in $D = 10$ have two different kinds of contribution, it becomes natural to separate the two, and allow them to be located at two different values of the radial coordinate r in the space transverse to the D3-brane. In fact it would seem now to be *unnatural* to insist that the two contributions be superposed at the same spatial location, since the position of the D3-brane is a modulus of the solution. Furthermore, the solutions will be supersymmetric for any choice of this modulus.

Now that the five-dimensional “positive-tension Randall-Sundrum brane” is recognised as corresponding to a negative-tension D3-brane in $D = 10$, the whole question of the possible instability of the model raises its head again. Also, one may wonder what happens concerning the trapping of gravity. These are two rather different questions, and the latter seems to be an easier one to address. We made a simple analysis of the gravity trapping, by studying the form of the wave-function for the massless four-dimensional graviton in the Randall-Sundrum limit, after having first separated the D3-brane and Z_2 -identification sources as described above. We found that the wavefunction tends to peak on the positive-energy Z_2 -identification surface, and to be suppressed on the negative-tension D3-brane. However, if the two singular surfaces are not too far separated, then the binding effect on the Z_2 -identification surface extends far enough to encompass the D3-brane region too. In this sense, gravity is still trapped in the vicinity of the two surfaces.

The question of stability, alluded to above, is less clear cut. On the one hand, the full one-parameter family of solutions with arbitrary placement of the D3-brane source is supersymmetric. This would seem to indicate that no instabilities can be present. Moreover, the full field theory clearly has a Bogomolnyi’i bound for the energy. On the other hand, a

negative-tension D3-brane considered in isolation would appear to have “ballooning modes” exploiting the fact that the energy is decreased if the area increases. The resolution of the apparent conflict between these two viewpoints remains unclear.

So far, we have refrained from speculating about any physical interpretation for the Z_2 -identification singular sources. We have treated them for now as arising via some *deus ex machina*, without attempting to give the associated surfaces any dynamical interpretation.⁴

A more serious attempt at providing an interpretation for the Z_2 -identification sources might be the following. From a type IIB perspective, what is needed is an object that carries no five-form charge, but that nevertheless folds up the transverse space. While we are unable to identify such a source in the maximally symmetric $\text{AdS}_5 \times S^5$ case, we can speculate on a possible interpretation in a case where there is a reduced symmetry in the transverse space. In such a circumstance, it seems that D7-branes may serve as a source for singular Z_2 -identification terms in the energy-momentum tensor. An examination of (37) suggests that if the assumption of spherical symmetry were relaxed, one could wrap a set of D7-branes around a 4-cycle of the 5-dimensional compact space. The D7-branes would then contribute with a strength $-\frac{1}{2}\tilde{\kappa}$ to four out of the five components of \hat{T}_{ab} , so that the factor of $-\frac{2}{5}\tilde{\kappa}$ in (37) results from ‘averaging’ over all five internal directions. For a single wrapped D7-brane, the effective tension is given by⁵

$$\tilde{\kappa}(\text{single D7-brane}) = \left(\frac{R_5}{20}\right)^{5/2} \pi^{-3} c^{-3/4} V_{4\text{-cycle}} \tau_{D7}, \quad (55)$$

which may be compared with $\tilde{\kappa} = \sqrt{20R_5} c^{-3/4}$ coming from (38). Note in particular that the factor $c^{-3/4}$ is common to both expressions.

This in fact leads to the picture of a Randall-Sundrum realization in the spirit of [25], where the Randall-Sundrum geometry arises from a warped F-theory construction. In [25], it was claimed that the six dimensions transverse to the 3-brane may be viewed as being the base of an elliptically fibered F-theory Calabi-Yau (complex) 4-fold. Curvature of the base may be related to the number of D3-branes and D7-branes present as well as the topology of the 4-fold itself. The kinked ‘thin-brane’ solution of (1) would then be an orbifold limit of this F-theory construction. Of course, the curvature sources arise not from the orbifolding

⁴One might be tempted to view the Z_2 -identification energy-momentum sources as being provided by a mechanism in the spirit of the turtles featured in the opening chapter of Stephen Hawking’s *A Brief History of Time*. The universe, according to an elderly lady attending a talk by a distinguished cosmologist, is supported on the back of a giant turtle. Before the cosmologist could ask the obvious question, the lady quickly added “and it’s turtles all the way down.”

⁵When the internal space is no longer S^5 , the Ricci scalar R_5 would more properly be replaced by the average curvature of the space.

itself but rather from the D3-branes and D7-branes pinned to the fixed orbifold planes. There is nevertheless still a need for an orbifold, since one needs to match not only the delta-functions in the energy-momentum tensor but also in all the form-field equations of motion. In particular, a D7-brane would require a charge for the 1-form $d\chi$, which however is absent in the Z_2 -identified flat space. Thus this D7-brane charge would need to be cancelled by something else, and an orbifold charge might be a natural candidate for this.

If this proposal were correct then our view of Kaluza-Klein theories would have come full circle. Initially it was thought that extra dimensions had to be compact in order to ensure a proper four-dimensional behavior. Then Randall-Sundrum II proposed an abandonment of this long-held view by demonstrating how an appropriate warp factor in the metric is all that is needed for the binding of modes to the brane. But with the suggestion of [25] that the space transverse to the D3-branes is simply the base of an F-theory Calabi-Yau 4-fold (whether smooth or in the orbifold limit), one once again returns to the original picture that a Kaluza-Klein theory necessarily has a compact internal space.

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